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13. ABSTRACT (Maximum 200 words) This project has encompassed research in four areas of control theory, namely robust, fixed-structure control, identification, adaptive cancellation, and nonlinear control. The robust, fixed-structure control techniques have been implemented in a Matlab toolbox for control design. The identification techniques include recursive methods with guaranteed convergence for high-order linear systems. The adaptive cancellation methods provide disturbance rejection for systems with uncertain or time-varying disturbance spectra with minimal system modeling. Nonlinear techniques for stabilizing unbalanced rotors and for finite-time stabilization have been developed. Experimental implementation and validation of the control methods developed under this project was performed on several laboratory-scale testbeds, including an acoustic noise experiment and an unbalanced rotating shaft experiment.					
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# Robust, Nonlinear Feedback Control

## Final Report

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# 1 Objectives and Summary

This is the final report for grant number AFOSR F49620-95-1-0019. This project addressed a broad range of problems in robust, nonlinear, and adaptive control that have application to aerospace systems and Air Force objectives. Under this project we developed novel control techniques along with supporting computational algorithms.

An additional accomplishment of the project was the experimental implementation and validation of the control methods developed under this project. These laboratory experiments were invaluable in providing guidance for pursuing the most promising lines of research. These experimental activities thus provided significant enhancement to the original scope of the project.

Several graduate students in the Aerospace Engineering Department at the University of Michigan were supported in part by this grant. Drs. Ravinder Venugopal and Sanjay Bhat received their Ph.D. degrees from the University of Michigan in August 1997. Dr. Venugopal's research was in adaptive control theory, while Dr. Bhat's area was nonlinear control. Also associated with this grant were two AASERT grants. Grant F49620-94-1-0409 supported the research of Dr. James Akers in linear system identification, while grant F49620-93-1-0502 supported the research of Dr. Robert Bupp in nonlinear control. The results of these two researchers, which were reported separately as required by those contracts, is summarized below. Finally, the Principal Investigator was responsible for supervising the research of two Palace Knight students. Dr. Andrew Sparks is currently at Wright-Patterson AFB, while Dr. R. Scott Erwin is currently at Phillips Laboratory. Several additional students who have not yet completed their Ph.D. degrees were partially supported under this grant.

Our efforts have focused on the following four projects:

**Robust, Fixed-Structure Control.** We developed new techniques and supporting computational algorithms for robust, fixed-structure controller synthesis. A MATLAB-based toolbox was developed for robust control design in the  $s$ -,  $\delta$ -, and  $z$ - domains with general constraints on the controller structure and multiple performance criteria. This toolbox was applied to a noise control experiment.

**Identification.** We developed and demonstrated new techniques for linear system identification. These identification techniques were applied to an acoustic duct experiment to obtain models involving lightly damped modes. These identified models were used as the basis for fixed-structure and adaptive controller synthesis.

**Adaptive Cancellation.** We developed and demonstrated new adaptive cancellation techniques for noise and vibration suppression and for control of rotating imbalance. For noise and vibration suppression these algorithms were demonstrated experimentally with both tonal, multi-tone and broadband noise. For rotating imbalance these algorithms were experimentally implemented and were shown to adaptively compensate for the effects of mass imbalance. Applications include magnetic bearing control as well as control of unbalanced control moment gyros for spacecraft applications.

**Nonlinear Control.** We developed and experimentally demonstrated nonlinear control techniques for two nonlinear electromechanical systems, namely, the rotational/translational actuator (RTAC) and the electromagnetically controlled oscillator (ECO). These nonlinear systems were realized in the form of laboratory experiments, and nonlinear controllers were implemented. In addition, we developed new results for continuous finite-time stabilization.

During the three-year period of this grant papers were presented at the following conferences:

IEEE Conference on Decision and Control, Orlando, FL, December 1994. [13, 28, 46, 81, 82, 92, 107]

American Control Conference, Seattle, WA, June 1995. [3, 15, 22, 29, 30, 49, 52, 58, 66, 68, 87, 94]

IEEE Conference on Decision and Control, New Orleans, LA, December 1995. [69, 83, 85]

IFAC World Congress, San Francisco, CA, June 1996. [11, 44, 71, 74, 88, 95]

IEEE Conference on Control and Its Applications, Dearborn, MI, October 1996. [16, 23, 26, 60, 75, 99, 100]

American Control Conference, Albuquerque, NM, June 1997. [7, 8, 24, 34, 39, 40, 42, 64, 102]

IEEE Conference on Control and Its Applications, Hartford, CT, October 1997. [2, 63, 77]

The Principal Investigator presented a plenary lecture at this conference. The title of this talk was "From Robust to Adaptive and Beyond: Liberating Control from the Tyranny of Modeling." In addition, papers were to appear at the conference

IEEE Conference on Decision and Control, San Diego, CA, December 1997. [1, 9, 41, 76, 101]

and papers were submitted to the conference

American Control Conference, Philadelphia, PA, June 1998. [19, 32, 37, 62, 73, 78, 79, 103]

All of the results obtained under this grant have been extensively documented in journal and conference papers. Specifically, the papers [17, 18, 53, 55, 59, 67, 72, 47, 48, 54, 56, 57, 84, 86, 89, 96, 108] were published, the papers [21, 25, 45, 61, 65, 70, 91, 105] were accepted for publication, and the papers [10, 20, 27, 31, 33, 35, 36, 38, 43, 80, 90, 97, 104, 111] were submitted for publication. All of the published papers are available from the given sources, and all of the submitted and accepted papers are available from the Principal Investigator.

All of the original research objectives have been achieved, and many of our accomplishments represent significant extensions of the originally proposed program. The Robust Fixed-Structure Toolbox has been distributed to several companies and universities. The identification and adaptive cancellation techniques have been experimentally implemented and tested on both laboratory scale experiments and an industrial facility. The nonlinear methods have been extensively developed and implemented on experiments as well. All of these results have been thoroughly documented in papers submitted or published in archival journals.

## 2 Technical Results

### 2.1 Robust Fixed-Structure Control

This effort has focused on the development of techniques for robust fixed-structure control. Under this grant we completed a fully portable version of a MATLAB-based Robust Fixed-Structure Toolbox. This toolbox has several unique features that give it capabilities that are not available from any other toolbox in the linear robust control area. These can be summarized as:

**1. Fixed-structure optimization via decentralized static output feedback controller architecture.** This approach to controller synthesis avoids both plant order reduction and controller order reduction by providing a direct path from a high-order plant model to a low-order controller. To provide a general formulation for fixed-structure controller synthesis we employ a decentralized static output feedback controller formulation. This problem formulation provides a unified controller framework for capturing arbitrary controller structures including static and dynamic controllers in centralized, decentralized, and hierarchical architectures with order constraints and arbitrary affinely parameterized state space realizations [44, 45].

**2. Robustness and performance measures.** To account for plant uncertainty, we developed new bounds for structured real and complex uncertainty. These bounds provide a vehicle for optimizing an  $H_2$  cost bound with respect to the controller parameters and can be used for structured singular value synthesis. In particular, scaled Popov bounds [14, 51, 86, 90, 91] provide upper bounds for the peak real structured singular value without frequency-dependent scales and multipli-

ers. Additional bounds include bounds based upon exclusion regions in the Nyquist plane [50, 55] as well as guaranteed cost bounds involving double commutators [98]. A new class of bounds is based upon a novel shift term which significantly reduces the conservatism of the cost bound. Shift terms have been developed for bounded-real-, positive-real-, and Popov-type bounds [56, 95, 97]. When used for controller synthesis [34, 35, 57, 64, 65] these bounds eliminate the need for frequency-dependent scales and multipliers. Alternative performance norms are considered in [39].

**3. Alternative time domains.** Although our previous research included both the continuous-time  $s$  domain and the discrete-time  $z$  domain, we have expanded this project to include controller synthesis in the  $\delta$  domain [40, 42]. The  $\delta$  domain can be viewed as an alternative parameterization of discrete-time dynamics wherein the dynamics matrix has the form of a perturbation to the identity matrix. With this structure, stable eigenvalues lie within a circle in the left half plane, whereas in traditional  $z$ -domain synthesis the unit circle is centered at the origin. This distinction is critical for plants involving lightly damped poles since, in standard  $z$ -domain synthesis, the poles tend to cluster near the point  $1 + j0$ , while in the  $\delta$  domain the poles lie near the origin. Consequently, numerical controller synthesis in the  $\delta$  domain is less sensitive than in the  $z$  domain. During this reporting period we developed techniques for fixed-structure mixed  $H_2/H_\infty$  control in the  $\delta$ -domain setting. Numerical comparisons have verified the advantages of  $\delta$ -domain discrete-time control as compared to the standard shift operator formulation. These techniques were applied to an acoustic duct experiment to design and implement robust low-order controllers [41].

**4. Quasi-Newton and Homotopy-Based Optimization.** To carry out fixed-structure controller synthesis we employ both homotopy and quasi-Newton optimization algorithms. Quasi-Newton algorithms are based upon available high-quality software, while homotopy algorithms have been developed in conjunction with Layne Watson of Virginia Tech [47, 48, 111]. To guarantee highly portable, machine-independent code we converted all FORTRAN programs to MATLAB in order to improve the reliability of the interfaces. This toolbox has been distributed to several researchers in industry and academia. Several researchers are actively extending the capabilities of the toolbox to include additional features.

## 2.2 Identification

The application of robust, fixed-gain control algorithms requires numerical plant models. Our laboratory experience has taught us the difficulty of obtaining reliable analytical models from physical principles due to hardware uncertainty. Hence, we developed identification methods for constructing plant models from measured data. Specifically, we developed a novel identification technique based upon *AR-MARKOV/Toeplitz models*, also known as predictive models in the adaptive control literature. This system formulation is neither time- nor frequency-domain based, but rather has the time-distributed input-output form of an ARMA model. Unlike ARMA

models, however, ARMARKOV models explicitly involve Markov parameters which, once identified, can be used to construct state space realizations using the eigensystem realization algorithm [4, 5, 6, 7, 8]. Convergence of the ARMARKOV/Toeplitz identification algorithm has been proven under a persistent excitation assumption.

Our numerical and experimental experience suggests that ARMARKOV models are fundamentally more resistant to noise than ARMA models. Specifically, sensor and process noise are always present in real data so that the plant model order is difficult to determine. Hence overparameterization is inevitable. In the presence of white noise inputs for identification, it is well known that a consistent estimate is guaranteed for only the feedthrough coefficient of the ARMA model. In fact, the inessential parameters of the overparameterized ARMA model are determined primarily by the measurement and process noise.

On the other hand, one can employ a high-order FIR model, in which case the numerator coefficients, which are precisely Markov parameters, can be estimated by least squares algorithms with consistency. However, long data records are required, and it is here that ARMARKOV models provide a clear advantage. In fact, it is easy to prove that consistent estimates of all of the Markov parameters in the ARMARKOV model are given by least squares methods, and simulations show that these estimates converge more quickly than the estimates provided by an FIR representation. In this way, ARMARKOV models provide improved identification in the presence of noisy data.

To demonstrate the ARMARKOV/Toeplitz identification algorithm, we developed a noise control experiment involving an acoustic duct. This experiment involves a disturbance source, two colocated microphone/control speaker pairs, and additional noncolocated microphones. The controlled system possesses numerous vibrational modes and thus provides a challenging testbed for both identification and multivariable robust control [59]. A multichannel spectrum analyzer and a control processor with automatic real-time code generation are used for data acquisition, identification, and controller implementation. By using measured time-domain input-output data, the ARMARKOV/Toeplitz identification algorithm was used to construct state space realizations encompassing up to 20 lightly damped modes of the acoustic dynamics. Both recursive and batch versions of the ARMARKOV/Toeplitz identification algorithm were implemented.

## 2.3 Adaptive Cancellation

The most significant accomplishment of this project is the development and experimental demonstration of an adaptive feedback controller for tonal and broadband disturbances. We believe that this new technique has the greatest potential for a significant impact on control engineering applications.

One of the main uses of feedback control is to suppress unwanted disturbances



which can cause excessive vibration levels and poor system performance. Disturbances can arise from a wide variety of sources. For example, rotating machinery can cause tonal or harmonic multi-tone disturbances, while turbulence can give rise to wide-band noise. The reduction of noise and vibration levels can be an important issue in aerospace vehicles. For example, aircraft engines can cause excessive noise and vibration, while helicopter blade motion can have a similar effect.

While fixed-gain controllers, such as those given by the fixed-structure methods discussed above, can be applied to vibration control problems, they are often cumbersome to apply in practice. In particular, fixed-gain methods require models of four distinct transfer functions, namely, the transfer functions from disturbance and control to performance and measurement. In practice, it is difficult to obtain these transfer functions, especially when the disturbance is distributed spatially and cannot be measured for identification purposes. In addition, changes in the plant dynamics and disturbance spectrum may necessitate extensive re-identification of the plant and redesign of the controller. Although robust control techniques can mitigate these difficulties somewhat, fixed-gain control techniques can be undesirable in the face of changing plant and disturbance conditions.

Although adaptive control is in general a difficult problem, the noise control community has developed a large class of algorithms that work in practice for a large class of systems of practical interest. These algorithms include LMS (least mean square) algorithms with FIR and IIR controllers, lattice filter techniques, and numerous variants. The theoretical foundation of these techniques varies greatly from method to method, as does their performance in practical application.

Under this grant we developed a novel adaptive disturbance cancellation technique based upon ARMARKOV/Toeplitz models and hence called *ARMARKOV/Toeplitz adaptive cancellation* [102]. This approach requires no prior knowledge of the disturbance spectrum and only minimal modeling of the controlled system's dynamics. Specifically, the transfer function from control to performance is required, although the remaining transfer functions (from control to measurement, that is, the plant per se, disturbance to performance, and disturbance to measurement) are not needed. These requirements are similar to those of other available adaptive cancellation techniques.

The mathematical basis for the ARMARKOV/Toeplitz adaptive cancellation algorithm is closely related to the ARMARKOV/Toeplitz identification algorithm. For applications the ARMARKOV/Toeplitz identification algorithm can be used to directly identify the control-to-performance transfer function (more specifically, the ARMARKOV/Toeplitz model of this transfer function) for use in the adaptive cancellation algorithm.

For validation, we implemented the ARMARKOV/Toeplitz adaptive cancellation algorithm on an acoustic duct experiment. Three classes of test disturbances were considered, namely, tonal, multi-tone, and white noise. For tonal disturbances, up to 40 dB suppression was obtained, while 10-20 dB was obtained for broadband



noise. This technique also performed similarly with broadband static generated by an AM radio tuner. The method demonstrated fast adaptation (less than 2 seconds) in the presence of changing disturbance spectra. A comparison of this algorithm with standard LMS algorithms is given in [77].

An analysis of feedforward versus feedback strategies in active noise control in [61] has been useful in understanding the problem of sensor/actuator placement. In particular, this paper provides a connection between control, measurement, disturbance, and performance colocation and the ability to suppress spillover in the closed-loop system. A further study of the relationship between feedback and feedforward control is given in [32].

State space analysis of systems with boundary input was studied as a basis for control design. Two problems were considered, namely, an acoustic duct with endspeaker [100] and slosh [99]. In addition, state space modeling of evanescent modes in acoustics was given in [80].

## 2.4 Nonlinear Control

Several nonlinear control problems were considered in this project. To address the problem of actuator saturation, which often is the most common nonlinearity to arise in an otherwise linear system, we developed synthesis techniques for full- and reduced-order controllers for systems with saturating actuators [12, 93]. These controllers, which provide asymptotic stability for a specified domain of attraction, do not restrict the control input to the interior of the constraint set but rather allow the control signal to “ride along” its boundary. The controllers are based upon bounded real and positivity criteria and allow both radial and independent multivariable saturation nonlinearities. The problem of rate saturation, which is important in flight control applications, was addressed as well in [96].

In another direction of research we applied integrator backstepping techniques to a low-dimensional nonlinear dynamics problem in [108]. This problem involves a device that can be viewed as a rotational/translational actuator (RTAC). Control of the RTAC provides a nonlinear benchmark problem which was the focus of an invited session held at the 1995 American Control Conference and is the basis for a special issue of International Journal on Robust and Nonlinear Control edited by the Principal Investigator [22].

To demonstrate our results we constructed an RTAC control experiment to provide a testbed for nonlinear control techniques. On this testbed we implemented dissipative and integrator backstepping controllers to guarantee stability and to study achievable performance [23, 25]. A novel class of controllers, motivated by the passive absorbers studied in [29], was developed specifically for this application but has broader applicability. Called *virtual resetting absorbers*, these controllers emulate the dynamics of passive absorbers except for the fact that the internal states are reset at

various times to reduce the “virtual energy” of the absorber. In this way, the control actuator is prevented from supplying energy to the system that would otherwise have been stored in a passive absorber. The theory of virtual resetting absorbers has been developed in [24, 27], while a specialized version, called the *virtual trap door absorber* is given in [26].

Hamilton-Jacobi theory was developed in [106] and applied to the problem of stabilizing the motion of a spinning top in both the symmetric and asymmetric cases [67, 109, 110]. The latter case, which corresponds to a mass imbalance, is of practical interest when the imbalance is not known. Since Hamilton-Jacobi theory fails in this case, we developed an *adaptive virtual autobalancing* technique that is able to stabilize rotating bodies with unknown mass imbalance [68, 70, 72]. Since this technique has potential practical value in rotating machinery, we fabricated two testbeds for studying the effects of mass imbalance, one based upon magnetic bearings and the other involving a controlled-gimbal platform. The former is applicable to current and future generation aircraft engine compressors, while the latter is applicable to control moment gyros for spacecraft control.

An additional topic in nonlinear control involves the study of non-Lipschitzian dynamics with application to controllers that yield finite-settling-time behavior. Such controllers efficiently use the available control authority with excellent overshoot properties. The underlying theory was developed in [15, 20], with applications to finite-time control of the double integrator [16, 21] and to Lagrangian dynamics [18].

To motivate the development of new nonlinear control methods, we also developed an experiment that involves stabilization of nonlinear systems. This experiment involves an electromagnet actuator to control a lightly damped oscillator. The experiment was constructed to serve as an analogue of a flexible mesh or membrane that can provide a large lightweight aperture for optical or RF applications. For this project we systematically explored a broad range of control strategies, including linear, nonlinear and adaptive methods. Our objective has been to develop techniques that are highly robust and require minimal prior modeling information. These results are described in [62, 63].

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